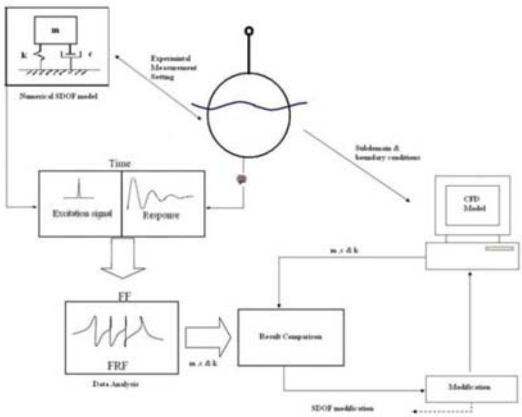
Master's Degree Thesis

ISRN: BTH-AMT-EX--2009/D-03--SE

Modelling of Buoyancy and Motion of a Submerged Body



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Dedicated to

I dedicate this thesis to my adorable wife, Nazi for all her love, patience and encouragement whiteout her efforts I would never finish my thesis and to my parents for their never-ending support and the sense of security they have given when I wanted it most, and to everyone who learned me thinking.

Mohammed Fadaee

Some things are better conveyed through share silence, and this is one such ..... to my parents for their love and their faithful unconditional support. I dedicate this thesis to them and to my brother Mohammed for his support. To my brothers, sisters and friends whose had been there when needed them.

Ahmedelrayah Masaad

I would like to dedicate this work to my parents ,the most kind , supportive and always stand by me. Also and I would like,to dedicate this work for all my friends who care about me and give me reasons to feel alive, and for every person who helped me in study and work.

Hazzaa Osman

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Thesis submitted for completion of Master of Science in Mechanical Engineering with emphasis of Structural Mechanics at the Department of Mechanical Engineering, Blekinge Institute of Technology, Karlskrona, Sweden.

Abstract

Modeling the dynamic of a submerged body has received a wide attention in the recent years. The trend is toward modeling the buoyancy and motion of the submerged body considering different variables affects on it. The scope of this thesis is modeling the dynamic of a submerged body by estimating its natural characteristic, m,c and k since it is necessary to design offshore structures and highly important in the ocean wave arena.

An experimental study is carried out to model the dynamic of a submerged body in its heave mode. Since extracting all parameters from this experimental is not an easy task when measuring low frequencies, a numerical model is developed. The resonance frequency (ω) , added mass (m_r) and the stiffness (k) of buoy is successfully estimated. The nonlinearity of the system is detected here by comparing the estimated dynamic characteristic parameters of a different amplitudes.

Keywords:

Submerged body, Dynamic characteristic, Hydrostatic stiffness, Added mass and damping, Modal test, COMSOL Multiphysics, SDOF

Acknowledgements

This thesis work was carried out at the Department of Mechanical Engineering, Blekinge Institute of Technology, Karlskrona, Sweden, under the supervision of Dr. Ansel Berghuvud. We would like to express our gratitude to our supervisor Dr. Ansel Berghuvud for his help, support and professional guidance during the thesis work, and we would like to thank PhD student Andreas Joesefsson for his contribution and advises in the experimental work and data analysis. It is a pleasure to thank the Karlskrona varmbadhus staff for giving us a chance to do our measurements in their swimming pool, also we deeply appreciate our colleagues, those who helped us to carry out the measurements in the swimming pool.

Karlskrona, June 2009

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Notations

A_w	water plane area $[m^2]$.
c	damping [N s/m].
F	force [N].
FE	Finite Element.
F_e	external force [N].
F_u	hydrostatic resorting force or the buoyancy force(F_b) [N].
F_r	radiation force [N].
f	frequency [Hz].
G_r	radiation impedance [N s/m].
g	gravity acceleration.
k	stiffness.
m	mass [kg].
m_f	fluid mass [kg].
m_r	frequency dependent added mass [kg].
R_r	frequency dependent added damping [N s/m].
r	radius.
$SDOF$	single degree of freedom.
$x,\ \dot{x}$ & $\ddot{x}$	displacement [m],velocity [m/s] & acceleration $[m^2/s]$.
ρ	density.
λ	system pole.
ω	damped natural frequency $[rad/s]$.
Ω	undamped natural frequency $[rad/s]$.
σ	damping factor $[rad/s]$.
ζ	damping ratio.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In recent years, the dynamics of a submerged body is of practical interest in the ocean engineering field as it is necessary for the submerged structures design . Moreover the new researches about generate a green energy from the ocean waves spot more of interest over this topic. Several studies are made on developing a mathematical and numerical modeling of the motions of a submerged body . (Daalan,1993 [1] - Tanizawa,1995 [2] & Wu &Tayler,1996 [3]) are developed a good technique to modeling the motion of a submerged body;that technique depends on the combination of Bernoulli equation with the equation of motions. (Bhatta &Rahman,2003) [4] determined the responses of vertical circular cylinder due to surge, heave and pitch induced by wave excitation and obtained the exciting force components. The limitation of those technique come from the limitation of the analytical model; their solutions are limited to specific applications of specific geometry and they will become difficult to apply when more design ideas is needed. Beside the analytical studies, many of modal tests are carried up to describe the motions of submerged body. Such experiment may not be possible at any stage of the design and for all geometries.

The present work attempts to estimate the dynamic characteristic parameters of a submerged body. It is studied the motion and buoyancy of a buoy with two different shape, spherical and cylindrical. Generally,the submerged body excited by the surface wave and experience six degrees of motion. In this work only the heave mode is considered. Beside an external force, the forces duo to the radiation and hydrostatics are considered. The work is included develop of a simplified numerical single degree of freedom (SDOF) and Finite Element (FE) model using MATLAB and COMSOL multiphysics software, respectively. Since the verification of those two numerical models is necessry an experimental measurements is carried out.

Experimentally, the stiffness, added mass and damping will be estimated. Furthermore, the numerical models will be developed to estimate those dynamic characteristic parameters. Since the step response technique will be used to extract the dynamic characteristic parameters from the experiment,the nonlinearity of the system will be observed as well. As it will be shown the submerged body damped very high compared to an equivalent non-submerged body and the results from the numerical methods is ,more or less, similar to the extracted results from the experimental measurement.

1.1 Aim and Objectives

The primary aim of this thesis is to model the dynamic of a submerged body in terms of its mass(m) , damping(c) and stiffness(k) parameters. A new experimental protocol beside a numerical model are developed in this thesis as a main part of its objective. This thesis object to set the basic concepts of the problem in order to use it in related project carrying in BTH and for further works. As part of this thesis, the knowledge of how to use COMSOL Multiphysics software to model this type of problem is developed.

1.2 Methodology

In order to achieve the aim of this thesis and apply its objectives, the project will follow below framework: (Fig. (1.1))

- An experimental measurement will be carried out using the modal test technique.
- Using MATLAB and COMSOL Multiphysics softwares, a numerical SDOF model will be developed based on the knowledge of dynamic of a submerged

body.

- A data will be collected from the literture, the developed numerical model and the experimental measurements.
- The collected data from the experiment measurement and the numerical model will be analyze to estimate m,c and k of the buoy and the results will be compare.

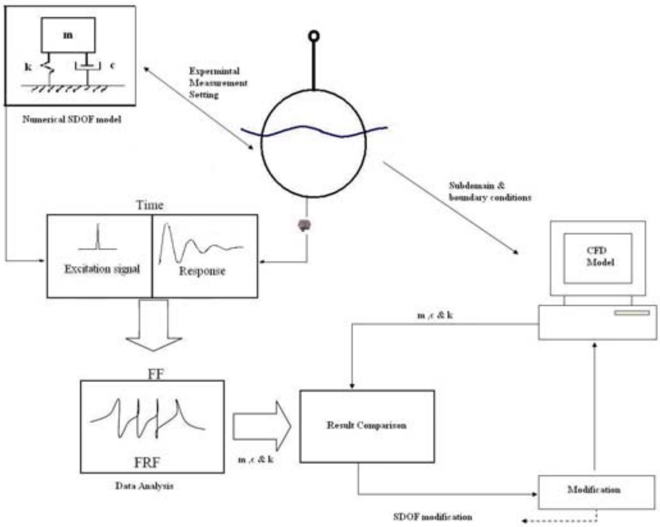

Figure 1.1: *Thesis framework.*

1.3 Research Structure

This report has been divided into five chapters presented as:

Chapter 1: introduction This chapter gives an overview about the thesis work. It is beginning with a brief background about modeling the dynamics of a submerged body. Then, it clarifies the aim and objectives of this research and highlight that methodology will be used to carry the project out.

Chapter 2: basic concept and theories This chapter review theories related to the dynamics of a submerged body; the hydrodynamics forces act on the submerged body and its motion due to that forces, buoyancy ,the added mass and the damping concept..etc. Also it reviews the concept of a single degree of freedom and how it is used to describe the motion of a submerged body.

Chapter 3 & **4: experimental investigation an numerical model** Those two chapters present our work on the experimental investigation and the numerical model. They are more involved in what we did and how we did it.

Chapter 5: Results and Discussion This chapter shows the obtained results from the experimental investigation and the numerical model and discuses those obtained results.

Chapter 2

Basic Concepts and Theory

2.1 Theoretical Model

For a proper understanding of the structure and the parameters influence on it, theoretical model is commonly used to describe the mechanical structures characteristic and the parameters influence. Generally, The concept of a single and multi degree of freedom (S-MDOF) system are used as the basic of the theoretical model in dynamic structure arena. Undoubtedly, the submerged body is more complicated than a SDOF system; though, with some assumptions it can consider as a single mass, spring and damper system. In this section the general case for a SDOF system will be used to model the dynamic of a submerged body.

2.1.1 Single Degree of Freedom System

Although very few structures could be considered as a single degree of freedom (**SDOF**) system , still SDOF concept is extremely important to develop the theoretical concept of vibrations. Schematically, SDOF system is representing as a mass connected to a spring and damper, Fig. (2.1). Where the mass is driven by a dynamic force F (t), and the damper and the spring are assume to be mass less.

Theoretically, SDOF system is expressed using the general equation of motion in time domain as [5]:

$$M\ddot{x} + C\dot{x} + Kx = F(t) \tag{2.1}$$

m = Mass of the body [kg]

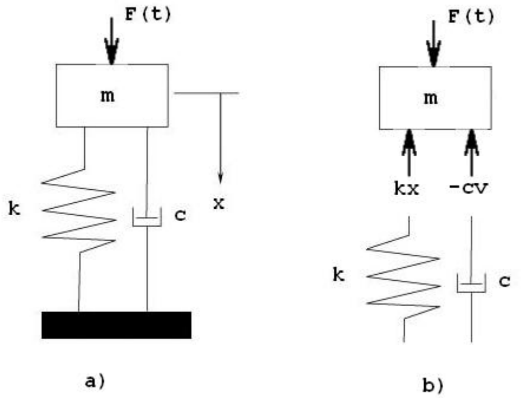

Figure 2.1: *SDOF system.*

$k =$ Spring stiffness, determined by how much force it takes to compress spring a unit of distance;($k = F_{spring}/x$) [N/m]

$c =$ Viscous damping constant, which is the ratio of the counter force provide by the damper to its velocity, ($c = -F_{dapmer}/v$)[Ns/m].

Considering the initial conditions are zero the solution of Eq.(2.1) in its Laplace or s transform can be written as:

$$(ms^2 + cs + k)X(s) = F(s) \tag{2.2}$$

leads to the roots, or the poles of the system λ. *(Randall J.,1994 for more information of how to convert to laplace and get the system modal parameters)*

From Eq. (2.2) we can observe that the system forcing F(s) is directly related to system response X(s),($X(s)/F(s)$), through the quantity $[ms^2 + cs + k]^{-1}$ which is known as the ***transfer function*** of the system. The transfer function is written as:

$$H(s) = \frac{X(s)}{F(s)} = \frac{1/m}{s^2 + (c/m)s + (k/m)} \tag{2.3}$$

Where denominator term of this equation is referred as the characteristic equation of the system.

$$s^2 + (c/m)s + (k/m) \tag{2.4}$$

with roots:

$$\lambda_{1,2} = -(c/2m) \pm \sqrt{(c/2m)^2 - (k/m)} \tag{2.5}$$

which simply can be written as:

$$\begin{aligned}\lambda_{1,2} &= -\Omega\zeta \pm j\Omega\sqrt{1-\zeta^2} \\ \lambda_{1,2} &= \sigma \pm j\omega\end{aligned}$$

$\zeta =$ Damping ratio, $\zeta = -\frac{\sigma}{\sqrt{\omega^2+\sigma^2}}$

$\omega =$ Damped natural frequency [rad/s]

$\sigma =$ Damping factor [rad/s]

$\Omega =$ Undamped natural frequency [rad/s], $\Omega = \sqrt{\omega^2 + \sigma^2}$

It is important to know that Eq. (2.3) is also known as a frequency response function (**FRF**) when represented in the frequency domain and it can be represented as residues (R) and poles (λ) as below:

$$H(s) = \frac{R}{s-\lambda} + \frac{R^*}{s-\lambda^*} \tag{2.6}$$

$* =$ Complex Conjugate

Where the poles give information about the system damping and natural frequency, while the residues give information about the mode shape of the system. Thus, by determining those parameters the dynamic characteristic of the system can be identified.

2.1.2 dynamic of a Submerged body

A submerged body is excited by the surface wave experience six degrees of motion; three translator and three rotational motions. Translator motions can be described in terms of heave, surge, and sway, while rotational can be described in terms of roll, yaw, and pitch, see figure (2.2) [6]. For simplification, we will investigate the body motion in z direction only since heave motion is important with respect to the oscillating of the submerged body.

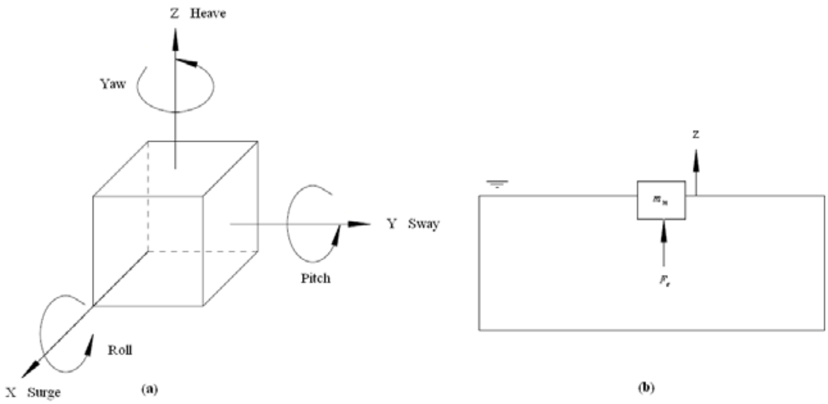

Figure 2.2: *Submerged Body: a/Translatory and angular displacements, b/ Motion in heave mode.*

In order to construct the governing equations of motion, the following assumptions are made:

1. Except in the heave motion, the body is not oscillate.
2. The submerged body is rigid with symmetric distribution of mass.
3. The viscosity and friction losses are neglected.
4. The mooring forces are not considered explicitly.

Let us assume that partially immersed mass ,(m_m), is excited by external force ,F_e, which resulting in a forced oscillatory motion represents by below equation:

$$(m_m)\ddot{z} = F_e \tag{2.7}$$

As known, structures show increase in damping and decrease in natural frequencies when immersed in the fluid. That change in the damping and natural frequencies is due to the influence of the surrounding fluid. When the oscillating body generates a radiated wave, the surrounding fluid exerts the system by the so-called reaction force F_r [7].This reaction force is represented as an added mass and damping. This added mass and damping can be written in the mechanical or radiation impedance terms G_r as: [8]

$$F_r = -G_r \dot{z} \tag{2.8}$$

where

$$\begin{aligned} G_r(\omega) &= R_r(\omega) + iX_r(\omega) or \\ &= R_r(\omega) + i\omega m_r(\omega) \end{aligned}$$

There is additional force acting on the oscillated body beside the radiation force. The oscillating in heave mode produces force named the hydrostatic resorting force F_u which is exerting on the fluid. This force is proportional to the body displacement from the equilibrium (z), $F_u \propto z$. The proportion's constant is so called the hydrostatic restoring coefficient (k). This resorting force can be written as below

$$F_u = kz \tag{2.9}$$

based on Archimedes principle:

> "Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object"

and the static-pressure term $\rho g z$, and from the equation of motion of the submerged body that shown in figure (2.3), the buoyancy equation can be written as:

$$F_b = \rho z_2 A - \rho z_1 A - W = \rho z_2 A - \rho z_1 A - \rho((z_2 - z_1)A - gV) = \rho g V \tag{2.10}$$

where F_b is the buoyancy force, ρ is the fluid density, A is the top area (AB) of parallelepiped and V is the volume of the submerged body under study. From the static (equilibrium) conditions: *the body force exerting on the fluid is equal to the fluid force exerting on the body*, we can see that left part of Eq. (2.9) and Eq. (2.10) are equal:

$$kz = \rho g V \tag{2.11}$$

in which we can see that the hydrostatic restoring coefficient is

$$k = \rho g A_w \tag{2.12}$$

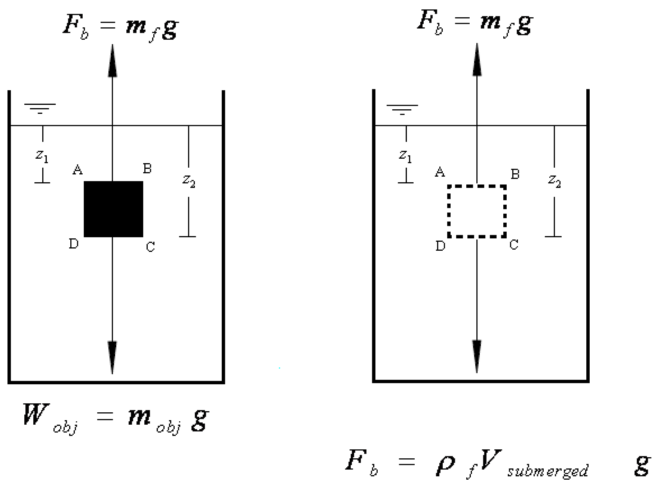

Figure 2.3: *Buoyancy force.*

where A_w is the body water plane area

$$[A_w(sphere) = \pi r^2(1 - z(0)^2/3r^2) \rightarrow A_w(cylinderical) = \pi r^2]$$

Under the above assumptions and considering the radiation and hydrostatic restoring force, Eqs. (2.8) and (2.9), in Eq. (2.7), the dynamic equation can be written as

$$(m_m + m_r)\ddot{z} + R_r\dot{z} + kz = F_e(t) \tag{2.13}$$

where m_r and R_r are the frequency-dependent added mass and damping coefficient, respectively.

This differential equation can be converted to its algebraic form using the Laplace transform. Considering zero initial conditions, an equivalent equation of dynamic Eq. (2.13) in Laplace or s domain can be written as below:

$$\begin{aligned} L\{M\ddot{z} + R_r\dot{z} + kz\} &= [Ms^2 + R_r s + k]Z(s) \\ L\{F_e(t)\} &= F_e(s) \end{aligned}$$

Thus Eq. (2.13) becomes:

$$[Ms^2 + R_r s + k]Z(s) = F_e(s) \tag{2.14}$$

or:

$$B(s)Z(s) = F_e(s) \tag{2.15}$$

$M = (m_m + m_r)$

$B(s)$ = system impedance, $B(s) = [Ms^2 + R_r s + k]Z(s) = 1/H(s)$.

Thus the system transfer function $H(s)$ can be written as

$$\begin{aligned} H(s) &= \frac{1}{B(s)} = \frac{Z(s)}{F(s)} \\ &= \frac{1/M}{s^2 + (R_r/M)s + (k/M)} \end{aligned}$$

where the denominator term is referred to as the characteristic equation of the system. From algebraic, the characteristic equation' roots are:

$$\lambda_{1,2} = -(R_r/2M) \pm \sqrt{(R_r/2M)^2 - (k/M)} \tag{2.16}$$

$$= (-\zeta \pm \sqrt{\zeta^2 - 1})\Omega \tag{2.17}$$

As mentioned in previous section, these roots are describing the dynamic behavior of the system , *submerged body*, in term of its damping and natural frequency.

2.2 Experimental Investigation

When it comes to trust finding theoretical model, verification and validation of that model are absolutely necessary. The common process used to describe the dynamic characteristic of a such submerged structure experimentally known as Modal Test or Experimental Modal Analysis. In this section, the general concept of that process and how it used to extract the natural characteristic of a structure will be shown.

2.2.1 Experimental Modal Analysis

Experimental Modal Analysis is an interesting experimental technique used in a wide range of practical applications such as identification of modal properties of structure, detection of structure using modal data, finite element model updating, active vibration control...etc. The purpose of this technique is to identify, understand, and simulate dynamic behavior and responses of structures. In other words,it is used to describe a structure in terms of its:

- natural frequency,
- damping, and
- mode shapes.

Experimentally, excitation mechanism and system response measurements are the most basic elements to measure in order to estimate the above parameters. However, Estimating the above parameters of a submerged body is of practical interest since it is necessary to design the offshore structures.

Exciter Mechanism There are two common excitation technique used in modal test; impact and shaker excitation. The first excitation technique is probably used when the structure is linear, and when data quality, and signal-to-noise-ratio have not that important effect on the expected result. The later excitation technique is widely used, specially when the higher data accuracy and good signal-to-noise-ratio are required. [9] The devices responsible in the impact and shaker excitation are known as impact hammer and shaker, respectively. The impact hammer is responsible to generate and apply an excitation force pulse to the test structure. While the shaker is responsible for generating a different type of excitation signal such as true random , pseudo random and burst sine signals with a wide range of frequency, dynamic, and amplitude. .However, the choice from those two excitation technique mainly depend on the measurement quality desired and the type of excitation system available. [10].

Generally,the geometry of the submerged body and the influence parameters are added more difficulty to use the shaker excitation technique. Because the shaker excitation technique is need to be attached directly to the structure, it seems no possibility to widely use it to excite the submerged body. On the other hand, the minimum required amount of hardware of the impact excitation and the portable property of the impact hammer consider as the best choice method for modal test of submerged body.

response measurements There are different types of transducers used in the modal test to measure the system response. Since measuring the displacement and the velocity of the system is not effective, measuring the acceleration is the most

Table 2.1: *Frequency response function formulation.*

Response	FRF
Displacement	Dynamic flexibility: x/F
Velocity	Mobility: v/F
Acceleration	Accelerance:a/F

common used method to measure the system motion. The most widely electromechanical sensor used for sensing motion is the piezoelectric accelerometer . When subjected to vibration, the piezoelectric accelerometer generates and outputs an electrical signal to the analyzer in the form of voltage. Sensitivity, dynamic range and the mass of the accelerometer are the factors should consider when selecting the acceleroemter.

2.2.2 Frequency Response Function

Frequency response function is the most common measurement of the modal test which expresses the system response to the excitation force. It can be formatting in different formulation such as dynamic compliance, mobility,and accelerance, since the system response is acquired in the displacement , velocity and acceleration format, respectively (Table 2.1).

Since the displacement and velocity are difficult to measure experimentally, normally the acceleration is measured beside the force and they represent in their frequency formulation to obtain the frequency response function (accelerance).

$$H_a(f) = \frac{A(f)}{F(f)} = -(2\pi f)^2 \frac{1/k}{1-(\frac{f}{f_0})^2 + j2\zeta\frac{f}{f_0}}$$

Note: Through integration and differentiation,it is easy to go from a formula to other.

To make sure the measured frequency response(accelerance) is reliable, there are some factors should be considered :

- Choose of a proper supporting method for a structure; that meet the analytical boundary condition and make sure it will not be changed during the test.

- Choose of a proper excitation system; that have a little influence on the desired measurement quality.
- Choose of a proper transducer by considering its resonant frequency, sensitivity, shock rating, mass and temperature range.
- Check of reciprocity, coherence , leakage, signal levels and linearity.

However, there are some process which can be used to improve the measurement accuracy:

- Averaging: use to increase the statistical reliability of a measurement and reduce the nonlinearities effects.
- Windowing: It is used to reduce the leakage effects.
- Increasing Measurement Resolution: It increases the closely spaced modes beside reduce the leakage. [11]

When the frequency response function is measured in a proper way, the system characteristic in terms of its resonance frequencies (f_0) and damping (ζ) can be determined, consequently.

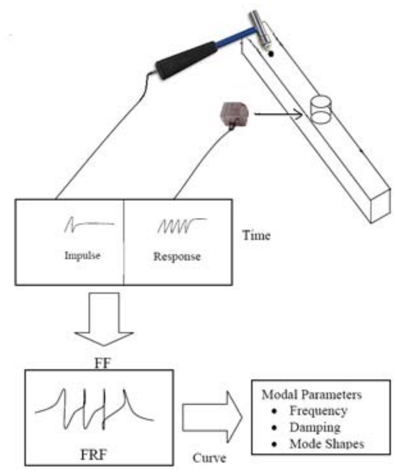

Figure 2.4: *General impact test configuration.*

Step Response: Step response is an alternative technique to estimate the modal parameters. In this method the system is unforced and at least one of its initial value is not zero. Let us begin with the dynamic equation of the submerged body of zero external force:

$$(m_m + m_r)\ddot{z} + R_r\dot{z} + kz = 0 \tag{2.18}$$

Assume ($\dot{z}(0) = 0$),The representation of above equation in the laplace domain is as follow

$$L\{(m_m+m_r)\ddot{z}+R_r\dot{z}+kz\} = [(m_m+m_r)s^2+R_rs+k]Z(s)-(m_m+m_r)sz(0)-R_rz(0) \tag{2.19}$$

Thus,

$$[(m_m + m_r)s^2 + R_rs + k]Z(s) = [(m_m + m_r)s + R_r]z(0) \tag{2.20}$$

so

$$Z(s) = \frac{[(m_m + m_r)s + R_r]z(0)}{[(m_m + m_r)s^2 + R_rs + k]} \tag{2.21}$$

or

$$A(s) = \frac{[(m_m + m_r)s + R_r]z(0)}{[(m_m + m_r)s^2 + R_rs + k]}s^2 \tag{2.22}$$

compare this equation the impulse response function from the vibration literature, the transfer function can be written as below:

$$H(s) = \frac{[(m_m + m_r)s + R_r]z(0)}{[(m_m + m_r)s^2 + R_rs + k]} \tag{2.23}$$

or in frequency domain

$$\begin{aligned} H(\omega) &= \frac{[(m_m + m_r)j\omega + R_r]z(0)}{[-(m_m + m_r)\omega^2 + j\omega R_r + k]} \\ &= \frac{-R_r((m_m + m_r)\omega^2 - k) - j\omega((m_m + m_r)^2\omega^2 - (m_m + m_r)k\omega + 2R_r^2)}{(-(m_m + m_r)^2\omega^2)^2 + \omega^2 R_r^2}x(0) \end{aligned} \tag{2.24}$$

As known,the real part of FRF cross the zero at the damped natural frequencies (ω),So

$$\frac{-R_r((m_m + m_r)\omega^2 - k)}{(-(m_m + m_r)^2\omega^2)^2 + \omega^2 R_r^2}x(0) = 0 \Rightarrow (m_m + m_r) = \frac{k}{\omega^2} \tag{2.25}$$

From equation (2.22)

$$imag(A(\omega)) = \frac{[(m_m + m_r)\omega^3 k - (m_m + m_r)^2\omega^3 - 2R_r^2]}{(-(m_m + m_r)^2\omega^2 + k)^2 + \omega^2 R_r^2} z(0) \tag{2.26}$$

So,

$$R_r = \sqrt{\frac{-imag(A(\omega))(-(m_m + m_r)^2\omega^2 + k)^2 - (m_m + m_r)^2\omega^3 + (m_m + m_r)\omega^2 k}{imag(A(\omega)) * \omega^2 + 2}} \tag{2.27}$$

finally from equations 2.25 and 2.27

$$m_r(w) = \frac{k}{\omega^2} - m_m \tag{2.28}$$

$$R_r(w) = \sqrt{\frac{-imag(A(\omega))(-(m_m\omega^2 + k)^2\omega + k)^2 - (m_m\omega^2 + k)[m_m\omega^4 + k\omega^2 + k]}{imag(A(\omega)) * \omega^2 + 2}} \tag{2.29}$$

2.2.3 Parameters Estimation

Having acquired the frequency response function,the modal parameters of a structure can be estimated. In attention to estimate the mass(m),stiffness (k) and damping (c) of the system,log-log plot format of the frequency response and the Half-power bandwidth method can be used.

Log-log is a plot of the frequency response spectra in its logarithmic scale. The reason of using the logarithmic scale instead of the linear scale when displaying spectra is to make sure that not visible details from the frequency response, because of its large dynamic range, is not loosing. Fortunately, many curves become straight lines on the log-log plot of the frequency response spectra which are some how related to m, c, and k of the system. It is important to mention for that slopes of the straight lines before and after the resonance frequency are corresponding to the stiffness and mass of the system, respectively. Normally, just as easily as the frequency response is measured, the modal mass (m) and stiffness (k) of the system can easily be estimated using the log-log plot format [12].

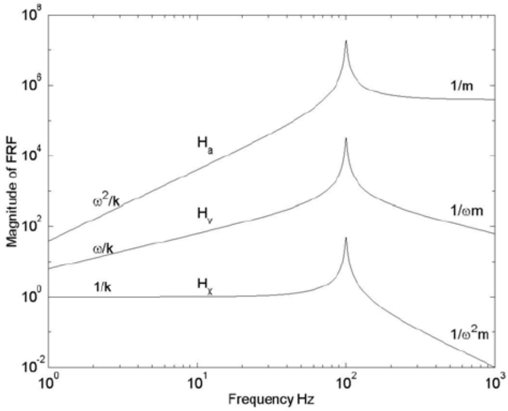

Figure 2.5: *log-log plot of the three common forms of frequency response, dynamic flexibility, H_x,mobility, H_v, and accelerance, H_a [Anders Brandat,2001].*

Because the damping of the system is an important parameter for modeling an object, there are some methods developed to estimate it. Half-power or the so-called 3dB bandwidth is one of that methods. It is often used to estimate the damping ratio (ζ) from the measured frequency response function. Basically,it depends on neglecting the influence of other modes on the mode under estimate. The Half-power bandwidth method is applied as follows [13]:

- Step one: Estimate the damped resonance frequency (f_{dr}) of each mode from the FRF plot; that correspond to the resonance peaks in Y axis.
- Step two: Identify the half power point(f_{1dr}&f_{2dr}); those correspond to the cross points of the straight line of slope ($y =\mid \hat{H}(f_{dr}) \mid \sqrt{2}$) with FRF curve, see figure (2.6)
- Step three: Estimate the damping ratio using below formula:

$$\zeta_r \approx \frac{f_{2dr} - f_{1dr}}{2f_{dr}} \tag{2.30}$$

note:It is very important to know that above formula is only valid for low damping.

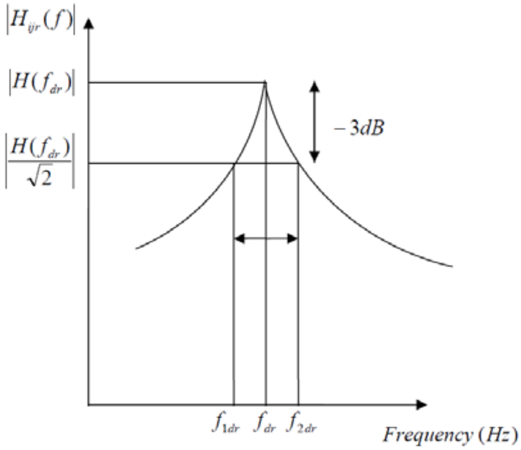

Figure 2.6: *Half-power bandwidth method [Lars Hkansson,2008].*

2.3 Finite Element Model

The finite element method is a numerical analysis (computational) technique which gives approximate solution to differential equation that model different kind of boundary problems in engineering and physics. In more and more engineering situations today, we find that it is necessary to obtain approximate numerical solutions to problems rather than exact closed-form solutions. On the other hand, sometime it is almost impossible to find the analytical solution Fig.(2.7). So use of the finite element method is extremely developing in engineering analysis and we can expect this use to increase significantly in coming years.

A finite element analysis is comprised of pre-processing, solution and post processing phases and it is same for all kind of problems like structural, heat transfer, fluid flow. And all finite element commercial softwares like COMSOL Multi physics follow these three steps.

2.3.1 Preprocessor

The goals of pre-processing are to develop an appropriate finite element mesh, assign suitable material properties, and apply boundary conditions in the form of restraints and loads which make the preprocessing (model definition) step critical. Every FEM

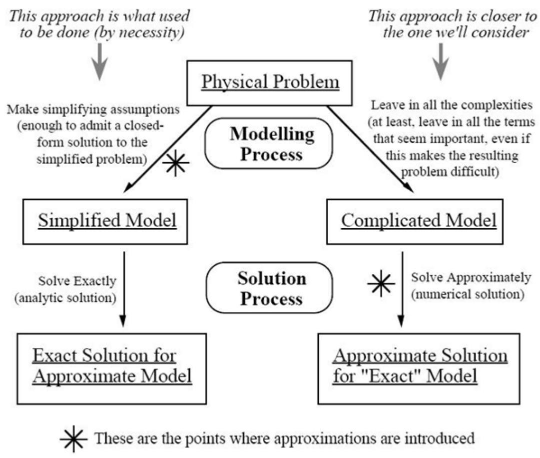

Figure 2.7: *Comparison between the numerical and the analytical solution process*

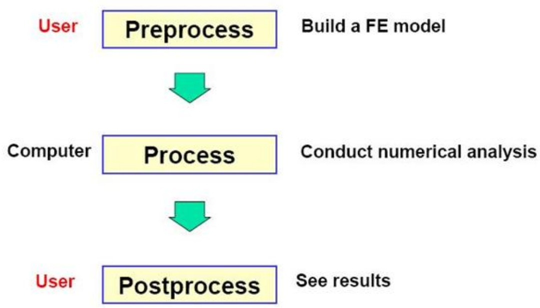

Figure 2.8: *Finite Element Modeling*

user will do the following steps to complete his model:

1. *Define and Mesh the geometry:* After defining the geometric domain of the problem we should mesh the continuum. The geometry is meshed with a mapping algorithm or an automatic free-meshing algorithm. The finite element mesh subdivides the geometry into those elements. The nodes, which are really just point locations in space, are generally located at the element corners and perhaps near each mid side. A variety of element shapes may be used and the different element shapes may be employed in the same solution region. Although the number and the the type of elements in a given problem are matters of engineering judgment, the analyst can rely on the experience of

others for guidelines.

2. *Select Interpolation Functions:* The next step is to assign nodes to each element and then choose the interpolation function to represent the variation of the field variable over the element. The field variable may be a scalar, a vector, or a higher-order tensor. Often, polynomials are selected as interpolation functions for the field variable because they are easy to integrate and differentiate. The chosen degree of the polynomial depends on the number of nodes assigned to the element, the nature and the number of unknowns at each node and certain continuity requirements imposed at the nodes and along the element boundaries.

3. *Define the material properties of the elements*

4. *Assemble the Element Properties to Obtain the System Equations:* To find the properties of the overall system modeled by the network of elements we must assemble all the element properties. In other words, we combine the matrix equations expressing the behavior of the elements and form the matrix equations expressing the behavior of the entire system.

5. *Define the Boundary Conditions:* Before the system equations are ready for solution they must be modified to account for the boundary conditions of the problem. At this stage we impose known nodal values of the dependent variables or nodal loads.

2.3.2 Solution

While the pre-processing and post-processing phases of the finite element method are interactive and time-consuming for the analyst, the solution is often a batch process and is demanding of computer resource. The governing equations are assembled into matrix form and are solved numerically. The assembly process depends not only on the type of analysis (e.g. static or dynamic), but also on the model's element types and properties, material properties and boundary conditions. As it is not uncommon for a finite element model to be represented by tens of thousands of equations, special solution techniques are used to reduce data storage requirements and computation time.

2.3.3 Postprocess

After a finite element model has been prepared and checked, boundary conditions have been applied, and the model has been solved, it is time to investigate the results of the analysis. This activity is known as the post-processing phase of the finite element method. Post-processing begins with a thorough check for problems that may have occurred during solution. Most solvers provide a log file, which should be searched for warnings or errors and which will also provide a quantitative measure of how well-behaved the numerical procedures during the solution. Once the solution is verified to be free of numerical problems, the quantities of interest may be examined. Many display options are available, the choice of which depends on the mathematical form of the quantity as well as its physical meaning.

Chapter 3

Experimental Measurement

The experimental work is a significant part of the science which gives the data to be analyzed to confirm or to drive the theorems. Modal analysis science is related to many kind of science such as signal processing, dynamic behavior, vibration and transducer technology. Our experimental works purpose is to estimate FRF for the submerged body and then estimate its dynamic properties; m, c and k. Therefore, we need to excite the structure and get the response, the structure consist of a buoy with additives such like plate to hit on and a mass to make the buoy more stable in the water. On the other hand the system is expected to have a low frequency vibration behavior, so accelerometers are used to estimate the input and the output signals.

3.1 System Overview

The impulse and step response are estimated for two kind of buoys while there were suspended in the operation situation. For the impulse response we used impact mass to fit the very long distance of the excitation point from the ground. Data acquisition device is connected to two accelerometers (in the impulse excitation) one of them in mounted with wax to the impact mass and the other one is mounted with wax to the buoy plate. For the step excitation only one accelerometer is used to measure the output , which connected to the buoy plate.

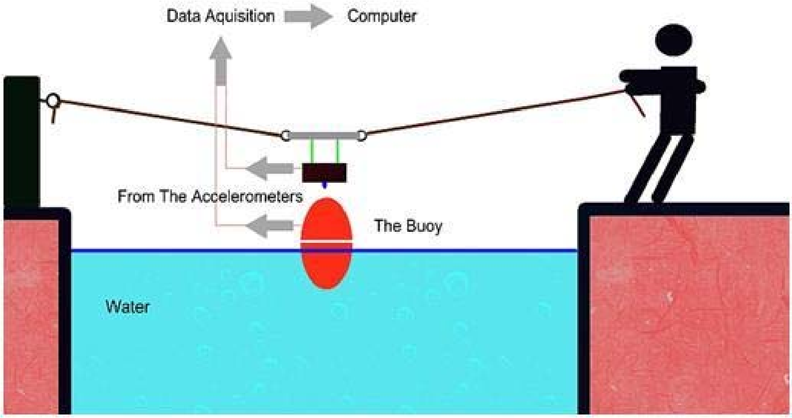

Figure 3.1: *Experimental system overview.*

3.1.1 Experiment Setup

Two kind of excitation were used in the experimental work; the first one is impulse hammer excitation which is done by the special hammer we made and the second excitation is the step excitation.

Structure Suspend The structure is simply suspended in operating condition, in the middle surface of deep water in the swimming pool; the only affect to the structure is the buoyancy force. Despite this kind of suspend may cause some difficulties in the theoretical model because of the displaced water, but we are interest in the interaction between the water and the structure after excite the structure, which generates a low frequencies vibration.

The impact mass There is a necessity to design special excitation method because of our measurements are in the middle of a swimming pool and the water should stay calm without disturbances, therefore the new excitation method should be able to excite our system from outside the pool. Some points are considered here:

- *The tip:* in each buoy there is a plate on the top to fix the accelerometer and that to be used as a flat surface to hit on. This plate is made of stiff plastic.
- Handling the mass and hitting method: the buoys are located in the middle of

the swimming pool; the mass should be able to used from outside of the pool.

We tried different kind of masses and tips in the pre-measurements till the results became stable. The special impact mass method consists of a plate, mass, elastic ropes, non-elastic rope, tip and an accelerometer. The tip attached in the hitting point in the impact mass which is suspended with two elastic ropes in the plate. In order to let the mass hit the buoy, the mass should be oscillate vertically and then let it fall free to hit the goal. The equipment is guided by four steel cables which are tied in the four holes in the plate. Finally to measure excitation signal we fixed an accelerometer on the mass. impact mass method details are shown in figure (3.2).

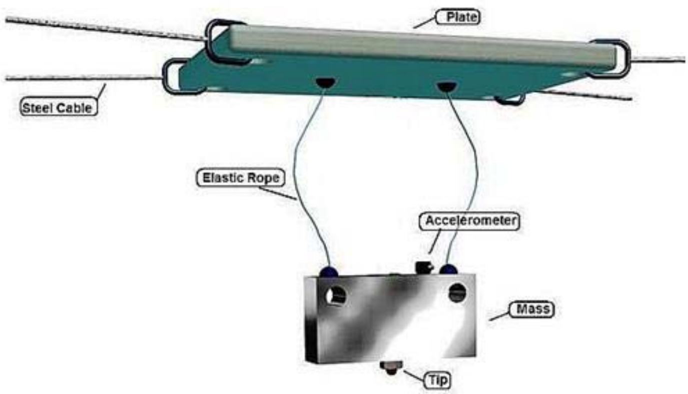

Figure 3.2: *Excitation method system.*

Test Object There are two kind of buoy are used in the experiment; the first one has a cylindrical shape and the second one is spherical. Since our aim is to hit the buoy in water we need a straight rigid enough surface. Consequently, we attached a plate on the top of the two buoys with different ways of fixing . The plate attached to the cylindrical buoy with two brackets, bolts and nuts joint as if appears in figure (3.3). On the other hand , in the spherical buoy there is no place to tie the plate with in the top surface, so we glued a tube to attache with the plate using brackets, bolts and nuts , shown in figure(3.3).

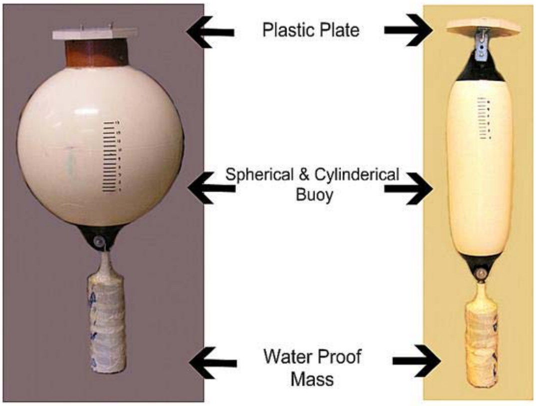

Figure 3.3: *Test object: cylindrical and spherical buoy of* $r = 0.1$ m & 0.213 m*, respectively.*

3.1.2 Measurement Equipment

These equipments are used in the measurements:

- Accelerometer KISTLER $8772A5$
- Accelerometer Delta Tron Type $4507B004$
- Data Acquisition Unit NI $cRIO - 9233$

Figure 3.4: *Accelerometer KISTLER* $8772A5$.

Figure 3.5: *Accelerometer Delta Tron Type* 4507*B*004.

Figure 3.6: *Data Acquisition Unit NI cRIO* $-$ 9233.

Chapter 4

Numerical Model

4.1 Single Degree of Freedom System Model

Based on the dynamic equation (Eq. 2.13), we can simulate the motion of a submerged body. First of all we have to get the values of $(m + m_r)$,R_r and k.

The hydrostatic stiffness (k) and the mass (m) are constant and independent of the frequency.The hydrostatic stiffness (k) can be calculated using equation (2.12) and the mass of the body is known. The frequency-dependent added mass m_r and damping R_r coefficient can be calculated using Havelock's dimensionless formula:

$$m_r(\omega) = m\mu. \; . \; . \; R_r(\omega) = m\omega\varepsilon$$

where μ and ε are the dimensionless coefficients of added mass and damping, respectively. Using Havelock's graph that shown in figure (4.1), we can estimate the dimensionless coefficients of added mass and damping. We may estimate the added mass m_r and the damping R_r by using the iteration as follow:

- Assume an initial value of damping frequency (ω).
- Use the initial value of damping frequency (ω) to estimate the dimensionless coefficients of added mass μ. It is done by estimating the value of μ_{33} that correspond to kr from Havelock's graph.
- Calculate the added mass coefficient $[m_r(\omega) = m\mu]$
- Use the added mass coefficient to estimate the damping frequency.

- Check the new value of the damping frequency with the initial assumed value.
  - If they are same, so you get the correct value of added mass and you can estimate the corresponding added damping dimensionless coefficient ε_{33} to estimate the added mass coefficient $[R_r(\omega) = m\omega\varepsilon]$.
  - Otherwise, use the new value of the damping frequency as a new guess and repeat the same processes till get the same result,approximately.

It is highly recommended to use MATLAB as a mathematical tool to simulate this single degree of freedom model. Fourth order Runge-Kotta method is used here to simulate the SDOF model based on below system of equations:

$$\begin{aligned} \frac{dz}{dt} &= v \\ \frac{dv}{dt} &= -\frac{k}{m+m_r}z - \frac{R_r}{m+m_r}v \end{aligned}$$

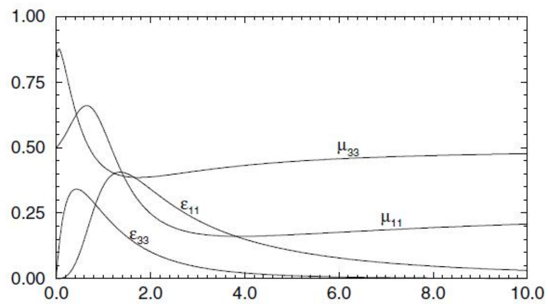

Figure 4.1: *Havelock's dimensionless surge and heave coefficients of radiation resistance and added mass [Johannes Falnes, 2004].*

4.2 Finite Element Model

COMSOL Multiphysics has been used to produce the Finite element model. We decided to do the simulation with the fluid interaction structure axial symmetry module.

Application modes and modules used in this model: Geom1 (Axial symmetry (2D))

- Axial Symmetry, Stress-Strain (MEMS Module)
- Moving Mesh (ALE)
- Two-Phase Flow, Laminar, Phase Field (MEMS Module)

Then we chose 2D axial symmetry model for our simulation. Obviously the most important advantage of this model compare with 2D model is that we can simulate the buoys which are spherical and cylindrical.

For making the buoys more stable we added 12kg mass under the buoys. Now for insert the effect of the mass in our model we can have the sphere and cylinder then we should increase the densities of the buoys in account of consider the mass. On the other hand to have more accurate results we can simulate the buoys with the hanging mass so we can have the real shape of the whole structure.

Table 4.1: *dimensions of the test objects.*

Specification	Spherical	Cylindrical
Radius(m)	0.213	0.1
Hight (m)	–	0.57
Weight(kg)	3.234	1.978
Density (kg/m^3)	80	110

We used a $10m * 4m$ domain geometry ($r = 10, z = 4$) for both spherical and cylindrical buoys.

Stability position:from Eq. (2.10)

$$F_b = m_f g = m_{obj.} * g = \rho_f g V_{submerged} \tag{4.1}$$

where $V_{submerged}$ is the buoy volume in the stability position.

- For the cylindrical:

$$V_{submerged} = (\pi * 0.1^2) * l = \frac{1.978 + 13}{1000}$$

So

$$l = 0.4767m$$

where l is the height of the surface area from the bottom of the cylinder.

- For the spherical: We have to use below formulas to calculate the volume of a submerged part of a spherical shape:

$$
\begin{aligned}
r &= (h^2 + r_1^2)/(2 * h) \\
V &= (\frac{\pi}{6})(3r_1^2 + h^2)h
\end{aligned}
$$

  Some how the stability position of the spherical buoy (d) was found about 0.046. m over the axial center as shown below.

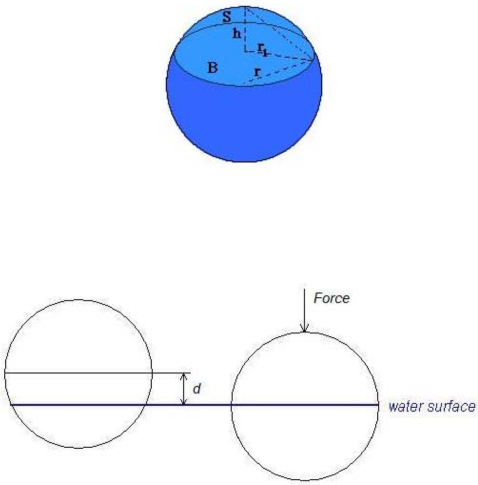

Figure 4.2: *Stability position (left) and the release amplitude(right)*

4.2.1 Sub-domain and Boundary Condition

When you use fluid-structure interaction module you will have two different domains called fluid and solid domain. In our model we have two solid domains for buoy and mass and two fluid domains for air and water. We have the water and air subdomain's characteristic like:

First we put -9.81 as the z gravity component in fluids subdomain but after solution we find that density in air and water is not fixed and instead of having just two different densities we have a distribution between 1.25 and 1000 in air and

-	Water	Air
Density	1000	1.25
Viscosity	$1e-3$	$2e-5$

water respectively. After solving the problem we found that we have positive phase field function, phi for the water and negative for the air subdomain. Therefore we can separate this two subdomain from each other. So we put the z and r gravity component equal to zero and instead set an volume force in z direction equal to $[-9.81 * if(phi < 0, 1.25, 1000)]$.

For the solid domain we just need to insert the gravity load per volume, $[-5416 * 9.81]$ for the mass, $[-80*9.81]$and $[-110*9.81]$ for the spherical and cylindrical buoy respectively. And finally we locked the buoy and mass in r direction to just have the vertical oscillating prevent it from turning or r direction movements.

We applied the fluid load for the buoy and mass boundaries those are in contact with the fluid in Axial Symmetry, Stress-Strain module boundary condition. and also we locked solid boundaries in moving mesh module in r direction. And finally we have the structural displacement for the solid boundaries in two phase flow module boundary conditions.

4.2.2 Mesh

Free meshing with triangular elements has been used for mesh the geometry and the maximum element size is 0.13. So we have the mesh statics like:

Number of degrees of freedom	62435
Number of triangular elements	6428
Number of boundary elements	340
Number of vertex elements	13
Minimum element quality	0.794
Element area ratio	0.001

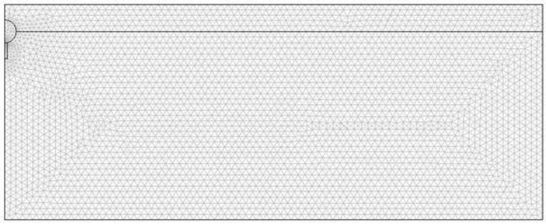

Figure 4.3: *Triangular element mesh*

4.2.3 Model Solution

Whenever you use the phase field two phase flow application mode, you need to perform two solving steps. First you only solve for the initial value for the phase field variables. you store the solution and switch the analysis type from transient initialization to transient. Then, you make a second calculation, using the stored solution as initial condition, when you want to solve it for all variables. But in both section we had the time dependent solver.

Chapter 5

Results and Discussion

5.1 Results of Experimental Investigation

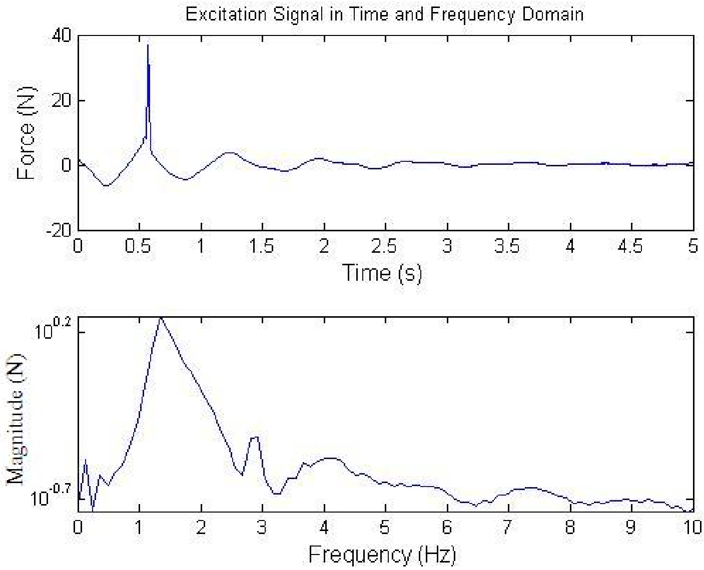

Figure 5.1: *Experiment excitation signal apply on a spherical buoy of* 0.213*m radius (Time and Frequency domain).*

Figs.(5.1-5.2) are shown that our experimental protocol gives a fairly expected data; the impulse force and the response.

From the FRF measurement, as shown in Fig (5.3), we can observe that peak

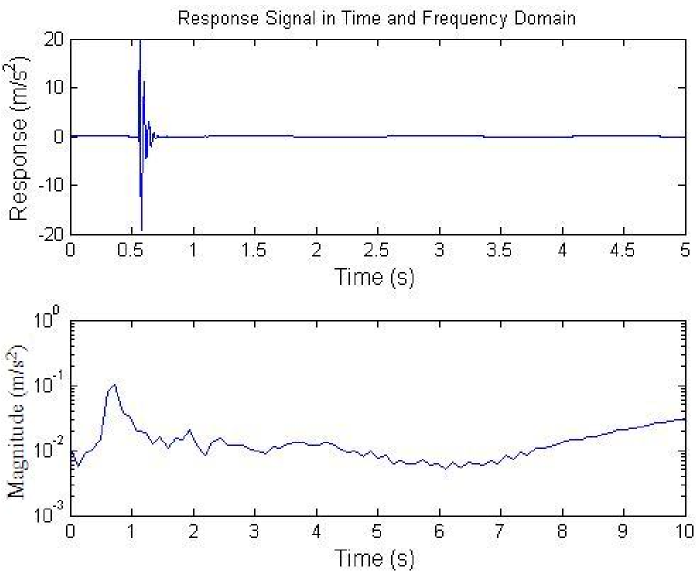

Figure 5.2: *Response of a spherical buoy of* $0.213m$ *radius (Time and Frequency domain).*

around $f = 0.7(Hz)$ which is the resonance frequency. Although, we can estimate the resonance frequency but still it is very hard to estimate the stiffness and added mass and damping of the system from the log-log plot of our frequency response measurement. Because the straight line after and before the resonance are not clear and the data experience a low noise-to-signal ratio around the resonance frequency. From the other hand, there is no possibility to improve our data by using any of the signal improvement technique. Authors think may one can success to estimate the modal parameters by conducting more data.

However, still the step response could be an alternative technique to estimate the modal parameters of a submerged body in low frequencies. From Fiqs. (5.5 - 5.6) we can see that the damping of the object is higher compared to expected result from non-submerged body of the equivalent dimensions and the spherical buoy is highly damping than the cylindrical one. This result direct us to think more about the effect of the submerged shapes and geometry on the dynamic characteristic of the submerged body.

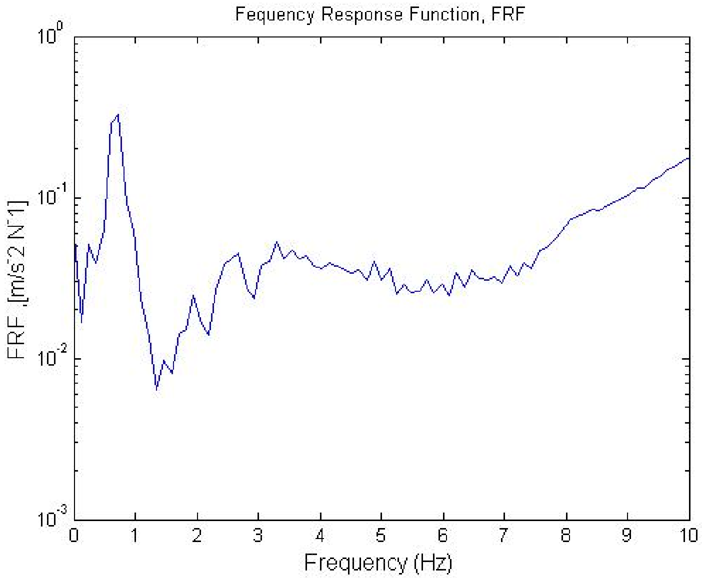

Figure 5.3: *Experimental frequency response function (FRF) of a spherical buoy of* $0.213m$ *radius.*

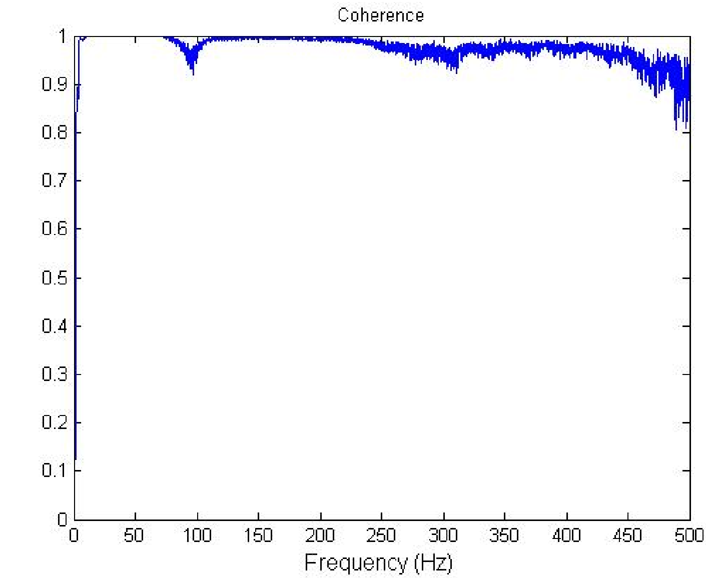

Figure 5.4: *Experimental Coherence function of a measured excitation and response of a spherical buoy of* $0.213m$ *radius.*

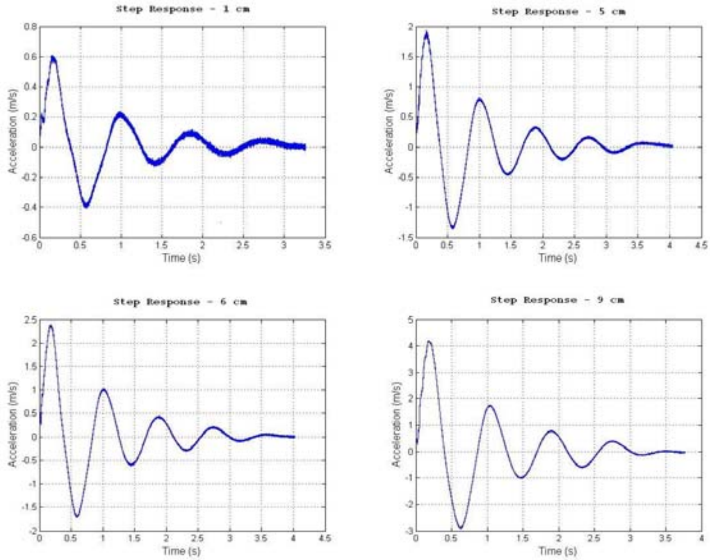

Figure 5.5: *Spherical buoy- Step response of different amplitude; 1,5,6 and 9 cm.*

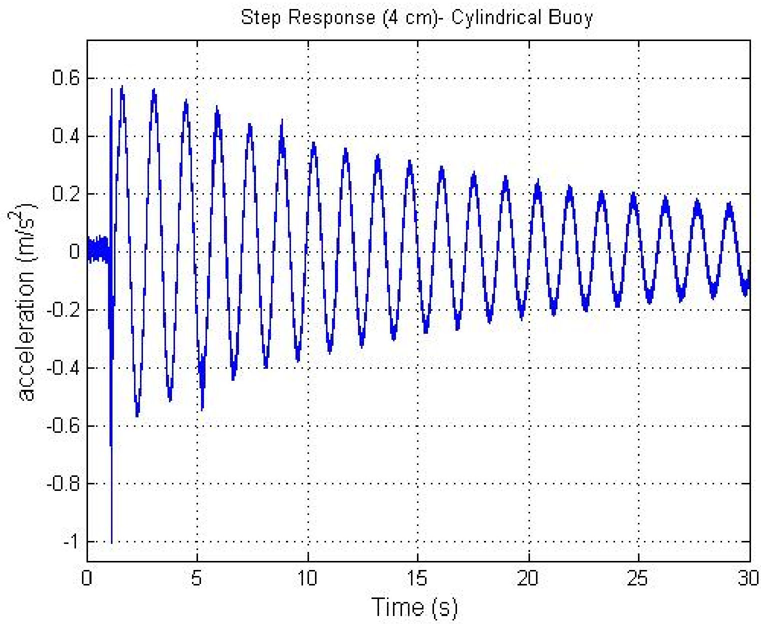

Figure 5.6: *Cylindrical buoy- Step response of 5 cm amplitude*

Table 5.1: *Dynamics characteristic of a buoy - Experimental measurement.*

specification	damping frequency	Added mass	Stiffness	added damping
.	f(Hz)	m_r(kg)	k(N/m)	R_r(Ns/m)
Spherical				
$x(0) = 1cm$	1.22	7.55	1397.2	$\star$
$x(0) = 5cm$	1.22	7.13	1372.5	$\star$
$x(0) = 6cm$	1.22	6.93	1361.24	$\star$
$x(0) = 9cm$	1.22	6.15	1315.02	$\star$
Cylindrical				
$x(0) = 4cm$	0.716	0.2613	308.2	$\star$

In Table (5.1) the experimentally estimated added mass of four different initial value ($x(0)$) using the impulse response are listed. As can be seen from the table the increase in added mass and the stiffness with increase in the step amplitude ($x(0)$) is highly observed. Clearly,we can observe the cylindrical shape is recorded very low added mass. Therefore, authors strongly believe the added mass is more depended on the water plane area rather than other factors.

5.2 Results of the Numerical Models

Table 5.2: *Dynamics characteristic of a buoy ($x(0) = 5cm$) - Numerical model.*

Model type	damping frequency	Added mass	Stiffness	added damping
	f(Hz)	m_r(kg)	k(N/m)	R_r(Ns/m)
$Literature(SDOF)$	1.23	6.33	1372.5	22.6
$FEMmodel$	1.14	9.8	1372.5	23

Different models in COMSOL with different geometries, kind of mesh and characters for both buoys have been tried . The numerical estimated added mass and damping listed in table (5.2) are extracted from the FE model using the SDOF curve fitting (Fig. 5.7). And from the the numerical SDOF model based on the the numerical values of non-dimensionalised heave coefficients of radiation resistance and of

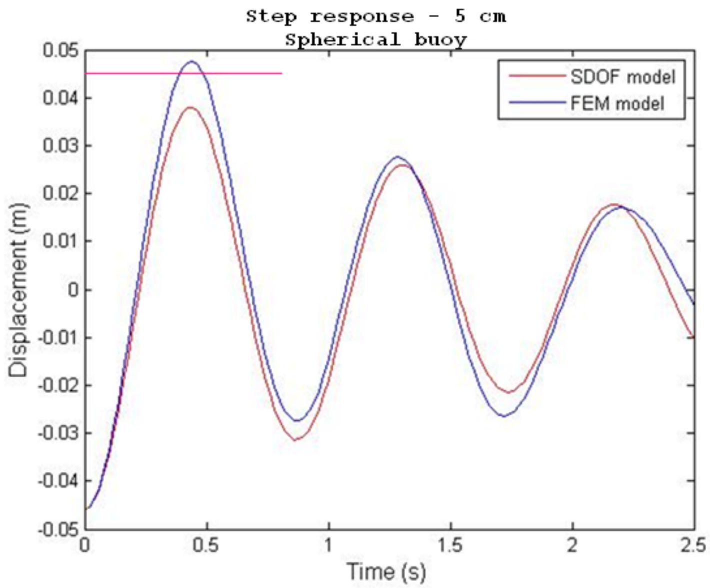

Figure 5.7: *SDOF curve fitting.*

added mass from Havelock [14]. As can seen from table (5.2) the result doesn't show a large similarity as expected. The reason of these differences seen in the results is due to the ignoring the nonlinearity in the SDOF model and accuracy of the used numerical methods (runge kutta versus Finite element method). From the results we can observe that the SDOF model is relatively close to the experimental result ,table (5.1) .But unless the nonlinearity coefficient is considered the model will be limited when more design ideas are required. From the other hand, the result from the FE model is not in good agreement with the experimental results. Authors may return that to two main reasons:

- first of all the model we simulate in COMSOL is not exactly same as the real structure that we had in swimming pool and we couldn't simulate the real one because we had some problem with the convergence during solution process. The buoy in COMSOL is completely spherical and there is no distance between the mass and the buoy in that model while the real buoy is not a complete sphere and we have a distance in the real structure between the buoy and additive masses.

- Secondly we don't have two different values of density for air and water and as mentioned before we have it as a distributed range between 1.25 (air) and 1000 (for water) and maybe this have an affect on the solution even when we solved this problem about the gravity and buoyancy force.

Authors believes by making some modification on this FE model, it can be useful to simulate the submerged body dynamics.

An interesting phenomena can be observe from fig. 5.7; which is the difference in the amplitude at the first peak. As we know from a SDOF concept, that system decay in amplitude has no possibility to have a peak with the bigger amplitude compare with the previous peak. But here we can see that the first peak cross the red line which show the distance that we release the buoy under the water. It means that buoy can travel a bigger distance when it coming out of water than it going downward into water with the same energy. It can be true then main reason is as the buoy coming out of water the area contacting with the fluid decrease so the resistance force due to the water will decrease then the buoy can travel more. On the other hand the curve could be wrong and differences refer to error coming from COMSOL. We think it is a good subject for the further works.

Chapter 6

Conclusions

In this thesis, an experimental measurements of estimating mass, damping and the stiffness of a cylindrical and spherical buoy are investigated. The new experimental protocol of exciting the buoy and carrying the measurement is made and verified. The data collected from three different sources: the literature, FE model and the experimental investigation. Based on the experimental results, the step response method is being suggested to estimate the dynamics of the submerged body instead of the frequency response measurements. While, the added mass, stiffness and the resonance frequency are estimated,the add damping is failed to be estimated. The FE model showed a possibility to estimate the m,R_r ,and k of the submerged body with a limited accuracy. As noted from the results, the submerged body damped very high compared to an equivalent non-submerged body and the system showed a nonlinearity. The system record unexpected phenomena; the buoy travel a bigger distance when it is coming out of water than it is going downward into water with the same energy.

In terms of modelling the dynamics of a submerged body it would be worth for further work to:

- Validate the FE model by simulate the model to be as same as the real structure.
- Expand the work to study the surge and sway modes and to to consider the wind and current waves.

- Study that phenomena observed here: Why the buoy travel a bigger distance when it coming out of water than it going downward into water with the same energy.
- Use another Finite Element software to model the same problem and compare the result with that from COMSOL Multiphysics. This comparison may give a good indication of which result is more trusted; the result from the literature, the result of the FE model or the experimental result.

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Appendix A

Appendix A

A.0.1 COMSOL Multiphysics

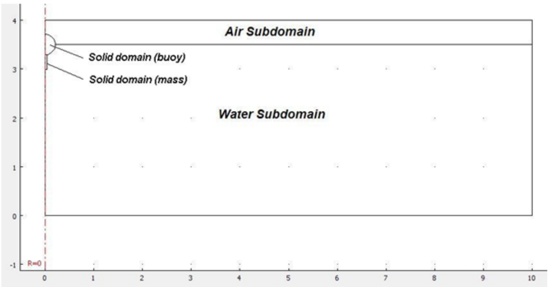

Figure A.1: *Different sub-domain for the whole structure.*

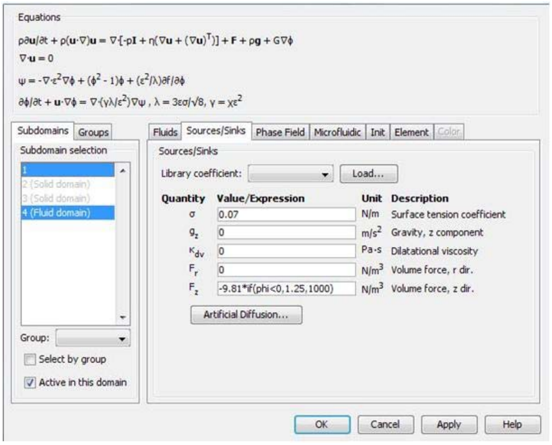

Figure A.2: *Sub-domain condition for the fluid domain.*

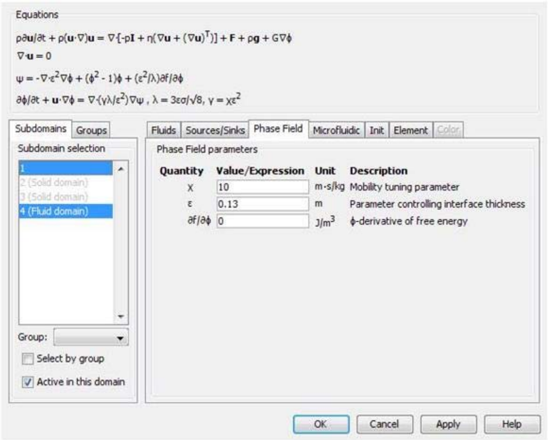

Figure A.3: *Sub-domain condition for the fluid domain.*

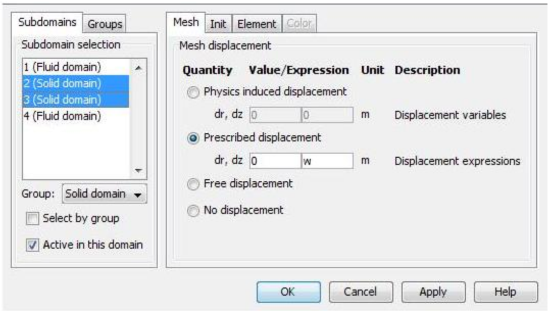

Figure A.4: *Solid sub-domain condition in moving mesh module.*

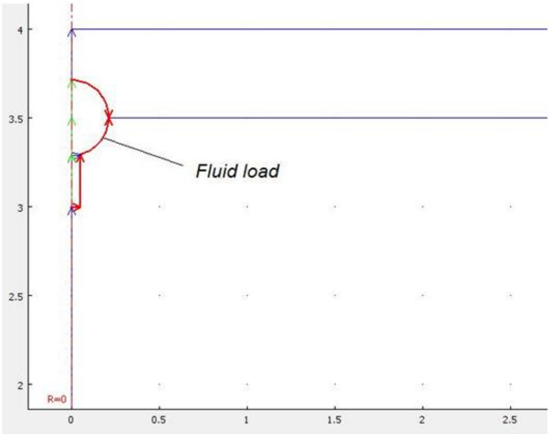

Figure A.5: *Boundary conditions in Stress-Strain module.*

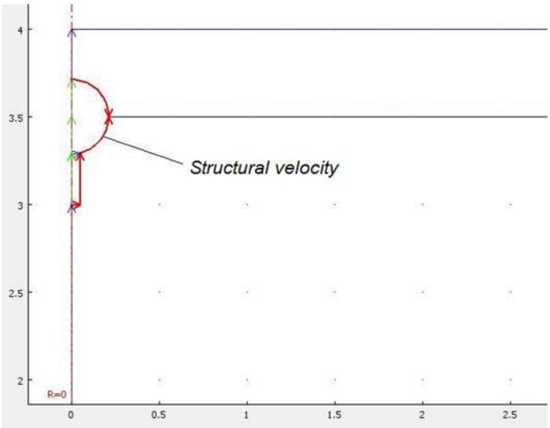

Figure A.6: *Boundary conditions in Two phase flow module.*

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